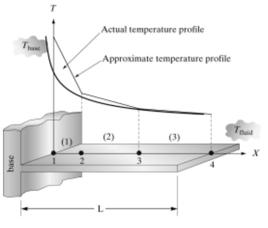
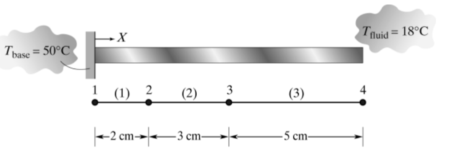
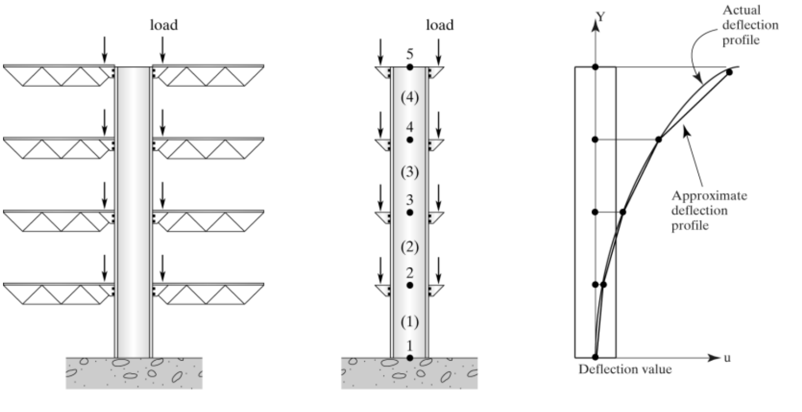
Chap. 3 One-Dimensional Elements





1-D Elements: A Column

- Columns: Commonly used to support loads from various floors of multi-story buildings.



1-D Elements: Linear Elements

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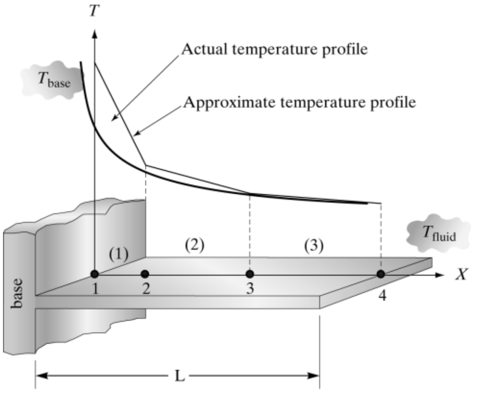
❑ 1-D Heat Transfer of a Fin

- Fins: Commonly used in a variety of engineering applications to enhance cooling
- Ex) Engine heads, heat sinks in electronic equipment, heat exchangers, etc.
- The temperature profile on a fin surface can be assumed as 1-dimensional (3 1-d elements)
- Linear temperature distribution

(1)



1-D Elements: Linear Elements

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❑ 1-D Heat Transfer of a Fin

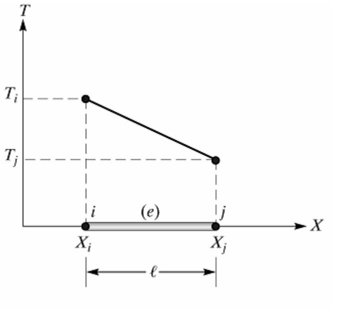
- The element's end conditions

$$X = X_i \rightarrow \quad T = T_i$$
$$X = X_j \rightarrow \quad T = T_j \qquad (2)$$

- Substituting (2) into (1)

$$T_i = c_1 + c_2 X_i \qquad T_j = c_1 + c_2 X_j \qquad (3)$$

$$T^{(e)} = \frac{T_i X_j - T_j X_i}{X_j - X_i} + \frac{T_j - T_i}{X_j - X_i} X$$



❑ Shape Function

- 1-d linear shape function

$$T^{(e)} = \underbrace{\left(\frac{X_j - X}{X_j - X_i}\right)}_{S_i} T_i + \underbrace{\left(\frac{X - X_i}{X_j - X_i}\right)}_{S_j} T_j$$

where

- Temperature distribution in terms of the shape f'ns

- Cf) For a structural example

$$u^{(e)} = \begin{bmatrix} S_i & S_j \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} \Rightarrow \Psi^{(e)} = \begin{bmatrix} S_i & S_j \end{bmatrix} \begin{Bmatrix} \Psi_i \\ \Psi_j \end{Bmatrix}$$

(general form)

1-D Elements: Linear Elements

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❑ Shape Function: Characteristics

- Has a value of unity at its corresponding node and has a zero at the other nodes (see the next slide in detail)
- In an element, the shape functions add up to a value of unity

- The sum of derivatives with respect to X is zero

1-D Elements: Linear Elements

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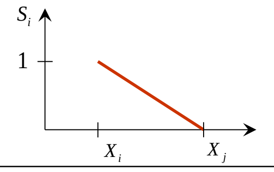
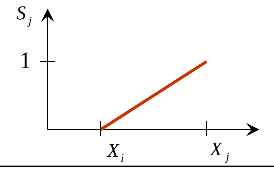
❑ Shape Function: Characteristics

	S_i	S_j
Node *i*	$S_i\big\vert_{X=X_i} = \frac{X_j - X}{\ell}\Big\vert_{X=X_i} = \frac{X_j - X_i}{\ell} = 1$	$S_j\big\vert_{X=X_i} = \frac{X - X_i}{\ell}\Big\vert_{X=X_i} = \frac{X_i - X_i}{\ell} = 0$
Node *j*	$S_i\big\vert_{X=X_j} = \frac{X_j - X}{\ell}\Big\vert_{X=X_j} = \frac{X_j - X_j}{\ell} = 0$	$S_j\big\vert_{X=X_j} = \frac{X - X_i}{\ell}\Big\vert_{X=X_j} = \frac{X_j - X_i}{\ell} = 1$
Shape		

1-D Elements: Linear Elements

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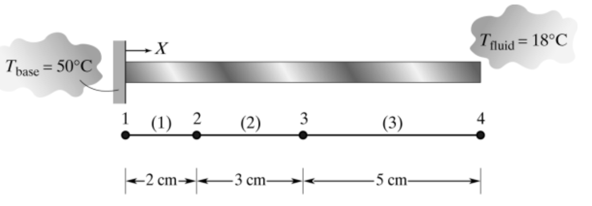
❑ Shape Function: Example

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 50 \\ 41 \\ 34 \\ 20 \end{Bmatrix} °C$$



- Temperature at X = 4 => element (2)

- Temperature at X = 8 => element (3)

1-D Elements: Quadratic Elements

❑ 1-D Heat Transfer of a Fin

- Quadratic approximation

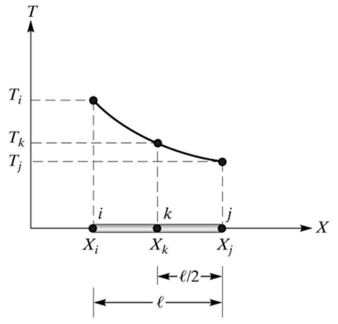
(4)

- The element's end conditions

$T = T_i$ at $X = X_i$

$T = T_j$ at $X = X_j$ (5)

$T = T_k$ at $X = X_k$



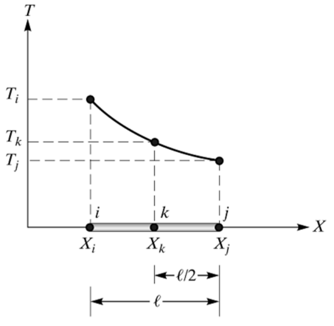
- Substituting (5) into (4)

$$T_i = c_1 + c_2 X_i + c_3 X_i^2$$
$$T_j = c_1 + c_2 X_j + c_3 X_j^2$$
$$T_k = c_1 + c_2 X_k + c_3 X_k^2$$

1-D Elements: Quadratic Elements

❑ Shape Function

- 1-d linear shape function



- Temperature distribution in terms of the shape f'ns

$$T^{(e)} = S_i T_i + S_j T_j + S_k T_k$$

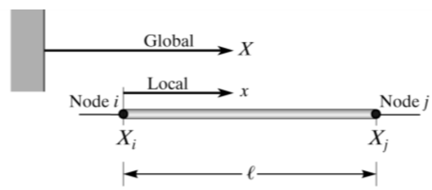
Coordinates: Global, Local, Natural

Global Coordinate System

- Needed to represent the location of each node and the orientation of each element
- Convenient to apply boundary conditions and loads (in terms of their global components)
- Solutions (eg. nodal displacements) is represented wrt. the global directions

Local Coordinate System

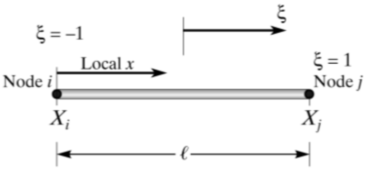
- Advantageous in computing integrals, especially the integrals contain products of shape fn‘s
- Usually started from 0: $0 \le x \le \ell$
- Shape fn‘s in terms of the local coordinates



Coordinates: Global, Local, Natural

Natural Coordinate System

- Local coordinates in a dimensionless form
- Normalized between -1 to 1:



- Linear shape f’ns in terms of the natural coordinate

Ex 3.3 (p. 133)

- Temperature distribution over an element of a 1-d fin

$$\xi = -1, \; T = T_i \qquad \xi = 1, \; T = T_j$$

- Transformation with other coordinates

Global coord. $X\left(X_i \le X \le X_j\right)$ Local coord. $x\left(0 \le x \le \ell\right)$

Coordinates: Global, Local, Natural

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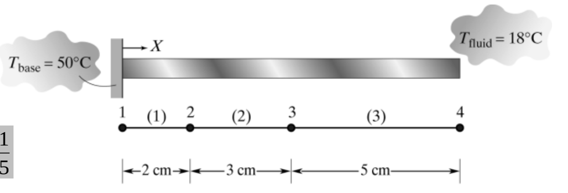
❑ Shape Function: Example

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 50 \\ 41 \\ 34 \\ 20 \end{Bmatrix} {}^\circ C$$

Temperature at X = 8
= > element (3)

$$\xi = \frac{2x}{\ell} - 1 = \frac{2\times 3}{5} - 1 = \frac{1}{5}$$



- Using shape functions based on the global coordinate
- Using shape functions based on the local coordinate
- Using shape functions based on the natural coordinate

Coordinates: Global, Local, Natural

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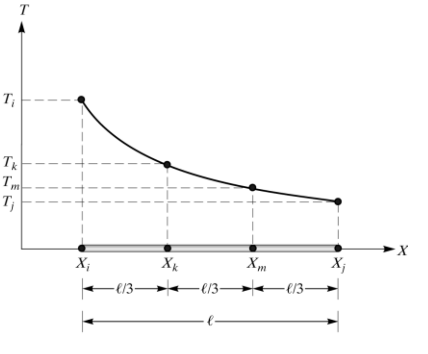
1-D Natural Quadratic Shape Function

1-D Natural Cubic Shape Function



❑ Gauss-Legendre Quadrature

- Integral: The sum of the product of certain weighting coefficients and the value of functions at some selected point (efficient in the natural coordinate system)

- Determination of the weighting coefficient :

$$x = c_0 + c_1\lambda \qquad [a, b] \Rightarrow [-1, 1]$$

$$a = c_0 + c_1(-1) \qquad b = c_0 + c_1(1)$$

$$x = \frac{(b+a)}{2} + \frac{(b-a)}{2}\lambda \qquad dx = \frac{(b-a)}{2} d\lambda$$

❑ Two-Point Gauss-Legendre Quadrature

- n = 2 => 4 unknowns $w_1, w_2 \quad \lambda_1, \lambda_2$

Ex 3.5~6 (p. 137~139)

- Legendre polynomials: $f(\lambda) = 1,\ \lambda,\ \lambda^2,\ \lambda^3$

$$w_1 f(\lambda_1) + w_2 f(\lambda_2) = \int_{-1}^{1} 1 d\lambda = 2$$

$$w_1 f(\lambda_1) + w_2 f(\lambda_2) = \int_{-1}^{1} \lambda d\lambda = 0$$

$$w_1 f(\lambda_1) + w_2 f(\lambda_2) = \int_{-1}^{1} \lambda^2 d\lambda = \frac{2}{3}$$

$$w_1 f(\lambda_1) + w_2 f(\lambda_2) = \int_{-1}^{1} \lambda^3 d\lambda = 0$$