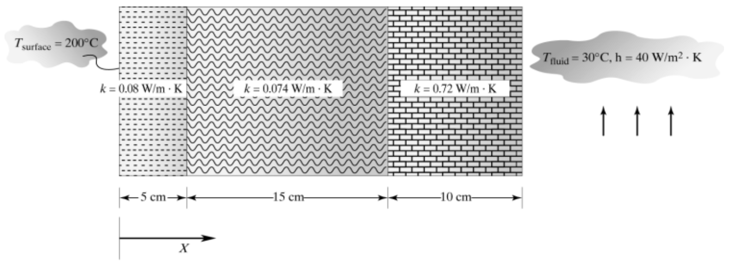
Chap. 4 One-Dimensional Analysis



[Review] Basic Steps in FEA

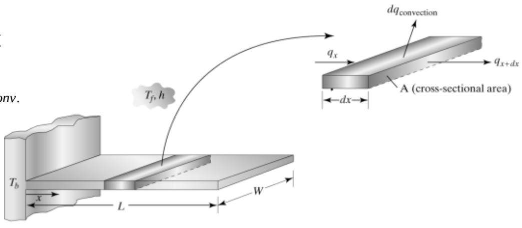
- **Preprocessing Phase**
  - Create and discretize the solution domain into finite elements (nodes & elements)
  - Assume shape functions to represent the behavior of an element
  - Develop equations for an element (Galerkin and min. potential energy approach)
  - Assemble the elements to represent the entire problem (global stiffness matrix)
  - Apply boundary condition and loading
- **Solution Phase**
  - Solve a set of linear equations simultaneously to obtain nodal results (DOF solution)
- **Postprocessing Phase**
  - Obtain other important information (usually, elementary information) : heat flux or stress in each element

Heat Transfer Problems

❑ 1-D Heat Transfer of a Fin

- Energy balance to a differential element

$$q_x = q_{x+dx} + dq_{conv.} \quad = q_x + \frac{dq_x}{dx}dx + dq_{conv.}$$



- Fourier's law

- Newton's law of cooling

$$0 = \frac{dq_x}{dx}dx + dq_{conv.} = \frac{d}{dx}\left(-kA\frac{dT}{dx}\right)dx + h(dA_s)(T - T_f)$$

- One-dimensional heat transfer equation

(p: perimeter, h: film coeff., T_f: surrounding temperature)

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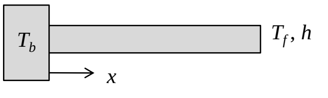
Heat Transfer Problems

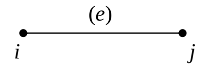
❑ 1-D Heat Transfer of a Fin

- Boundary conditions



The fin base	Temperature	
The fin tip	Temperature	
	Insulation	
	Heat loss (by convection)	

❑ Equations for an Element



- Temperature distribution for an element

where $S_i = \dfrac{X_j - X}{\ell}$ $\quad S_j = \dfrac{X - X_i}{\ell}$

- Generalization of the governing equation

$$-kA\frac{d^2T}{dX^2} + hp(T - T_f) = 0$$

The residual to be orthogonal to some weighting functions Φ_i

- Galerlkin residuals for an arbitrary element with node i and j

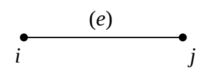
❑ Equations for an Element

- By applying the chain rule

$$\frac{d}{dX}\left(S_i\frac{d\Psi}{dX}\right) = S_i\frac{d^2\Psi}{dX^2} + \frac{dS_i}{dX}\frac{d\Psi}{dX} \quad \Rightarrow \quad S_i\frac{d^2\Psi}{dX^2} = \frac{d}{dX}\left(S_i\frac{d\Psi}{dX}\right) - \frac{dS_i}{dX}\frac{d\Psi}{dX}$$

- Galerkin equation + chain rule

$$R_i^{(e)} = \int_{X_i}^{X_j} S_i\left(c_1\boxed{\frac{d^2\Psi}{dX^2}} + c_2\Psi + c_3\right)dX = 0$$



❑ Equations for an Element – node i

- Integration of the first term: $S_i = 0$ at $X = X_j$ $\quad S_i = 1$ at $X = X_i$

- Integration of the other terms:

$$S_i = \frac{X_j - X}{\ell} \qquad \Psi = S_i\Psi_i + S_j\Psi_j = \frac{X_j - X}{\ell}\Psi_i + \frac{X - X_i}{\ell}\Psi_j$$

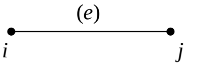
❑ Equations for an Element – node j

$$R_j = \int_{X_i}^{X_j} S_j^{(e)} \left[c_1 \frac{d^2\Psi}{dX^2} + c_2\Psi + c_3 \right]^{(e)} dX$$

$$\int_{X_i}^{X_j} c_1 \left(\frac{d}{dX}\left(S_j \frac{d\Psi}{dX} \right) \right) dX = c_1 S_j \frac{d\Psi}{dX}\bigg|_{X=X_j} - c_1 S_i \frac{d\Psi}{dX}\bigg|_{X=X_i} = c_1 \frac{d\Psi}{dX}\bigg|_{X=X_j}$$

$$\int_{X_i}^{X_j} c_1 \left(-\frac{dS_j}{dX}\frac{d\Psi}{dX} \right) dX = -\frac{c_1}{\ell}\left(-\Psi_i + \Psi_j \right)$$

$$\int_{X_i}^{X_j} S_j (c_2\Psi) dX = \frac{c_2\ell}{3}\Psi_i + \frac{c_2\ell}{6}\Psi_j$$



$$\int_{X_i}^{X_j} S_j c_3 dX = c_3 \frac{\ell}{2}$$

❑ Equations for an Element – Matrix form

$$\begin{Bmatrix} R_i \\ R_j \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -c_1 \left.\dfrac{d\Psi}{dX}\right|_{X=X_i} \\ c_1 \left.\dfrac{d\Psi}{dX}\right|_{X=X_j} \end{Bmatrix} - \frac{c_1}{\ell}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \Psi_i \\ \Psi_j \end{Bmatrix} + \frac{c_2\ell}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \Psi_i \\ \Psi_j \end{Bmatrix} + \frac{c_3\ell}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\overbrace{\begin{Bmatrix} c_1 \left.\dfrac{d\Psi}{dX}\right|_{X=X_i} \\ -c_1 \left.\dfrac{d\Psi}{dX}\right|_{X=X_j} \end{Bmatrix}}^{\text{boundary condition}} + \overbrace{\left\{[\mathbf{K}]^{(e)}_{c_1} \quad [\mathbf{K}]^{(e)}_{c_2}\right\}}^{\text{The elemental conductance or stiffness}}\begin{Bmatrix} \Psi_i \\ \Psi_j \end{Bmatrix} = \overbrace{\{\mathbf{F}\}^{(e)}}^{\text{Load matrix}}$$

❑ Equations for an Element – Matrix form

- Elementary conductance matrix

- The heat loss through the tip surface

$$\begin{Bmatrix} c_1 \left.\dfrac{d\Psi}{dX}\right|_{X=X_i} \\ -c_1 \left.\dfrac{d\Psi}{dX}\right|_{X=X_j} \end{Bmatrix} = \begin{Bmatrix} kA \left.\dfrac{d\Psi}{dX}\right|_{X=X_i} \\ -kA \left.\dfrac{d\Psi}{dX}\right|_{X=X_j} \end{Bmatrix} = \begin{Bmatrix} 0 \\ hA\left(T_j - T_f\right) \end{Bmatrix}$$

Heat Transfer Problems

❑ Equations for an Element – Matrix form

- The elementary conductance matrix

: w/o considering BC on the tip

: with considering BC on the tip

- The thermal load vector

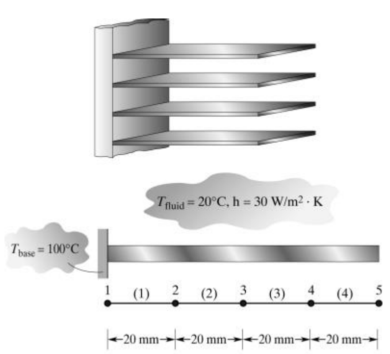
: w/o considering BC on the tip

: with considering BC on the tip

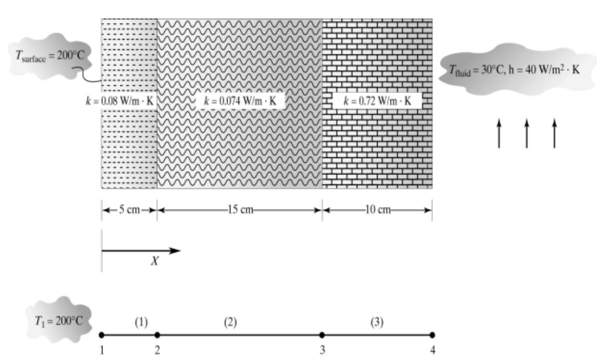
Heat Transfer Problems

❑ Examples (p. 152 ~ 158)

A fin problem



A composite wall problem



Solid Mechanics Problems

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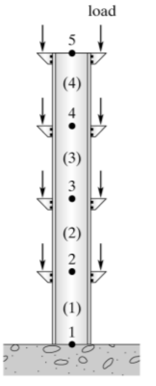
❑ A Column under Axial Loading

- Use of the minimum total potential energy formulation
- The strain energy for an element
- The total potential energy for the entire body

(For n elements and m nodes)

- The minimum total potential energy principle



Solid Mechanics Problems

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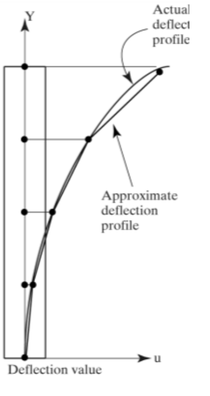
❑ Construction of an Elementary Stiffness Matrix

- Minimizing the strain energy with respect to u_i and u_j

$$u^{(e)} = S_i u_i + S_j u_j$$

$$S_i = 1 - \frac{y}{\ell} \qquad S_j = \frac{y}{\ell}$$

- The strain in each member (element)
- The strain energy for an arbitrary element (e)



❑ **Construction of an Elementary Stiffness Matrix**

- The deflection for an arbitrary element with nodes i and j

❑ **Construction of an Elementary Load Vector**

Ex 4.4 (p. 164 ~ 166)

- Minimizing the work done by external forces