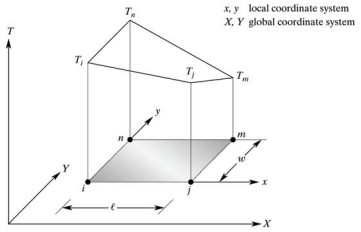
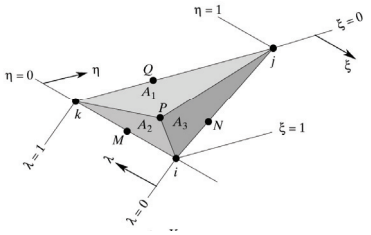
Chap. 5 Two-Dimensional Elements





2-D Elements: A Plate

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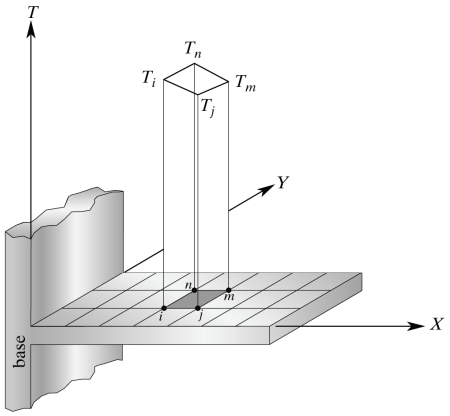
- 2D Plate: functions of X and Y



2-D Elements: Rectangular Elements

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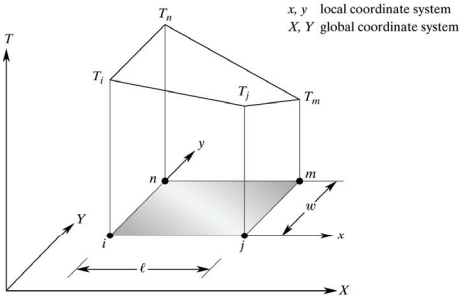
❑ 2d Shape function: Formulation

- Temperature distribution for an arbitrary rectangular element

$$T^{(e)} = b_1 + b_2 x + b_3 y + b_4 xy$$

- Nodal temperature

$$x = 0,\ y = 0 \Rightarrow T = T_i$$
$$x = \ell,\ y = 0 \Rightarrow T = T_j$$
$$x = \ell,\ y = w \Rightarrow T = T_m$$
$$x = 0,\ y = w \Rightarrow T = T_n$$



2-D Elements: Rectangular Elements

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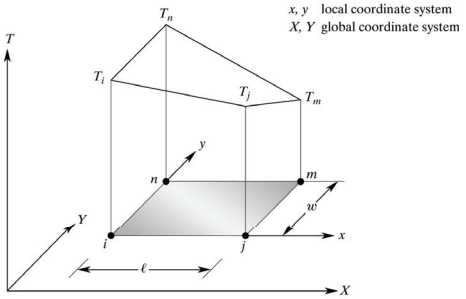
❑ 2d Shape function: Formulation

- Temperature distribution in terms of shape f'n
- The shape functions (in local coordinate)



2-D Elements: Rectangular Elements

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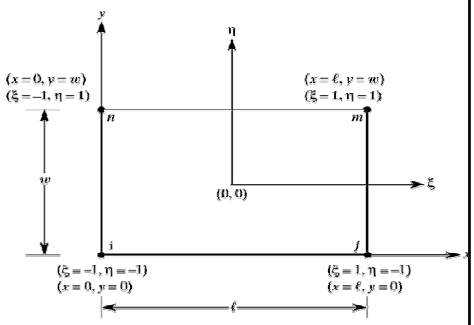
❑ Shape Function: Natural Coordinate

- Local coordinates in a dimensionless form [-1, 1]
- Suitable to the numerical integration by Gaussian quadratures
- Natural coordinates:
- 2d shape functions

Note: Check 3 characteristics of the shape functions



2-D Elements: Rectangular Elements

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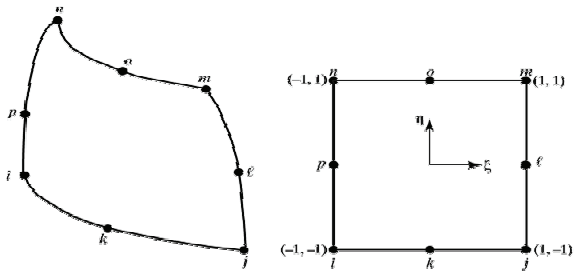
❑ Quadratic Quadrilateral Elements

- 8-mode quadratic elements: higher order version of the 2d 4-node elements
- Better suited for modeling problems with curved boundaries
- Consists of 4 corner nodes and 4 midpoint nodes (2nd-order interpolation)



2-D Elements: Rectangular Elements

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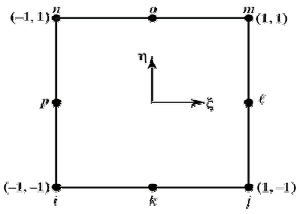
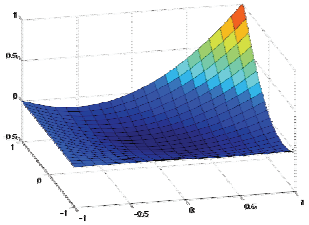
❑ Quadratic Quadrilateral Elements

- Shape functions for 4 corner nodes





2-D Elements: Rectangular Elements

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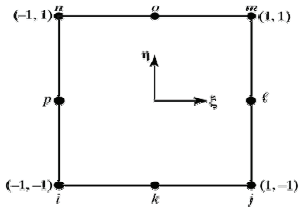
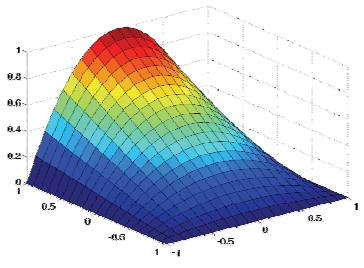
❑ Quadratic Quadrilateral Elements

- Shape functions for 4 midpoint nodes





Ex 5.1 (pp. 195 ~ 197)

2-D Elements: Triangular Elements

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❑ Linear Triangular Element: Formulation

- Temperature distribution in an element

$$T^{(e)} = a_1 + a_2 X + a_3 Y \qquad (1)$$

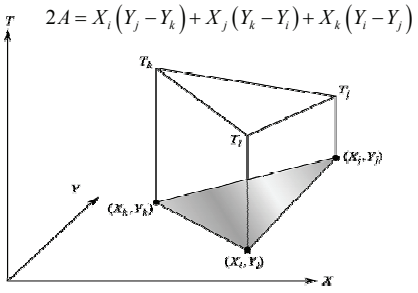
- Nodal temperature conditions

$$T = T_i \quad \text{at} \quad X = X_i \quad Y = Y_i$$
$$T = T_j \quad \text{at} \quad X = X_j \quad Y = Y_j \qquad (2)$$
$$T = T_k \quad \text{at} \quad X = X_k \quad Y = Y_k$$

- Substituting (2) into (1)

$$T_i = a_1 + a_2 X_i + a_3 Y_i$$
$$T_j = a_1 + a_2 X_j + a_3 Y_j$$
$$T_k = a_1 + a_2 X_k + a_3 Y_k$$

$$2A = X_i\left(Y_j - Y_k\right) + X_j\left(Y_k - Y_i\right) + X_k\left(Y_i - Y_j\right)$$



2-D Elements: Triangular Elements

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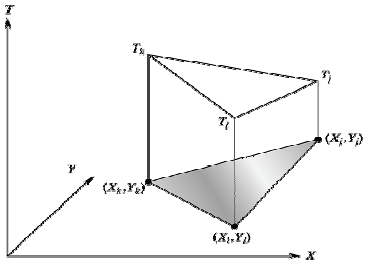
❑ Linear Triangular Element: Shape F'n

- Temperature distribution in terms of shape f'n

$$T^{(e)} = \begin{bmatrix} S_i & S_j & S_k \end{bmatrix} \begin{Bmatrix} T_i \\ T_j \\ T_k \end{Bmatrix}$$

- The shape functions (in local coordinate)



$$\alpha_i = X_j Y_k - X_k Y_j \qquad \beta_i = Y_j - Y_k \qquad \delta_i = X_k - X_j$$
$$\alpha_j = X_k Y_i - X_i Y_k \qquad \beta_j = Y_k - Y_i \qquad \delta_j = X_i - X_k$$
$$\alpha_k = X_i Y_j - X_j Y_i \qquad \beta_k = Y_i - Y_j \qquad \delta_k = X_j - X_i$$

2-D Elements: Triangular Elements

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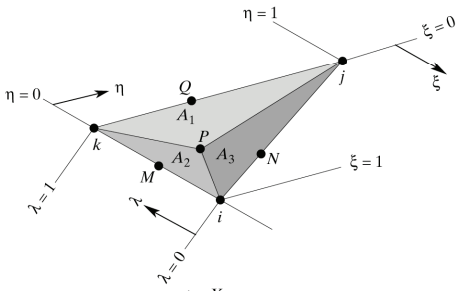
❑ Natural (Area) Coordinates

- 1-d linear shape function

$$\frac{A_1}{A}+\frac{A_2}{A}+\frac{A_3}{A}=\frac{A}{A}=1=\xi+\eta+\lambda$$



- Area coordinates vs. shape f'ns

$$\xi=S_i \qquad \eta=S_j \qquad \lambda=S_k$$

2-D Elements: Triangular Elements

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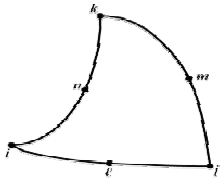
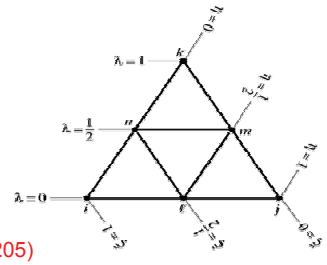
❑ Quadratic Triangular Element: Shape F'n

- Temperature distribution in terms of shape f'n

$$T^{(e)}=a_1+a_2X+a_3Y+a_4X^2+a_5XY+a_6Y^2$$

- The shape functions (in natural coordinate)





Ex 5.3 (pp. 202 ~ 205)

2-D Elements: Isoparametric Elements

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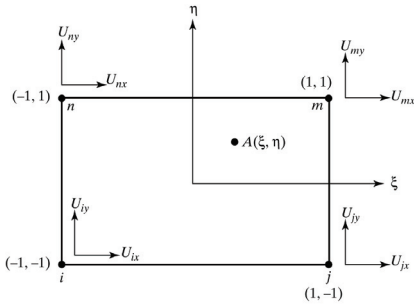
❑ Isoparametric Element

- Isoparametric: The same parameters (shape functions) are used to express the geometry and to define the unknown variables
- The displacement field within a quadrilateral element
- The position of any point within the element



2-D Elements: Numerical Integration

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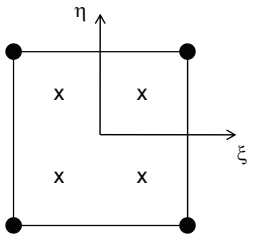
❑ 2-D Gauss-Legendre Quadrature Formulation

- Integration interval: [-1, 1] => Natural coordinate
- Cf) 2-pts Gauss-Legendre Quadrature in an 1-d element

$$w_1 f(\lambda_1) + w_2 f(\lambda_2) = \int_{-1}^{1} f(\lambda)\, d\lambda$$

$$w_1 = w_2 = 1 \qquad \lambda_1 = -\lambda_2 = -0.577350269$$

- 4-pts (2x2) Gauss-Legendre Quadrature in a 2-d element



ξ-dir:

η-dir: